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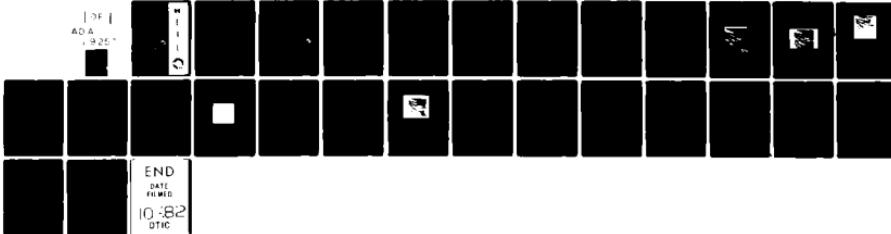
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AN ANALYSIS OF A RELAXATION SCHEME TO IMPROVE TERRAIN ELEVATION--ETC(U)

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An analysis of a relaxation
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Michael A. Crombie
James A. Shine

JULY 1982



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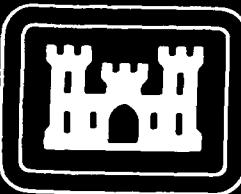
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PREFACE

This study was conducted under DA Project 4A762707A855, Topographic Mapping Technology.

The study was done during 1981 under the supervision of Mr. Dale E. Howell, Chief, Information Sciences Division, and Mr. Lawrence A. Gambino, Director, Computer Sciences Laboratory.

COL Edward K. Wintz, CE was Commander and Director and Mr. Robert P. Macchia was Technical Director during the report preparation.

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AN ANALYSIS OF A RELAXATION SCHEME TO IMPROVE TERRAIN ELEVATION DATA

INTRODUCTION

The Engineer Topographic Laboratories (ETL) has been concerned for some time about the fundamental concept of image registration for all mapping operations wherein stereointersection is a component function.^{1,2,3} It is the authors' belief that too much concern in the past has been placed on speeding up an inadequate process rather than on determining ways to improve the product. Since the output of the compilation process provides the basic structure, Digital Terrain Matrix (DTM), it is important for subsequent feature extraction that the DTM be determined accurately. A variety of in-house studies were performed to evaluate linear correlation and several of its close relatives.^{4,5,6,7,8} The work to date indicates that something other than conventional correlation is required if automated compilation is to become a reality.

¹Barbara A. Lambird, David Lavine, George C. Stockman, Kenneth C. Hayes, and Laveen N. Kanal, *Study of Digital Matching of Dissimilar Images*, Final Technical Report, ETL-0248, Contract No. DAAK70-79-C-0234, L.N.K. Corporation, 302 Notley Court, Silver Spring, MD, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA, November 1980, AD-A102 619.

²Michael A. Crombie, *Stereo Analysis of a Specific Digital Model Sampled from Aerial Imagery*, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA, ETL-0072, September 1976, AD-A033 567.

³F. Raye Norvelle, *Interactive Digital Correlation Techniques for Automatic Compilation of Elevation Data*, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA, ETL-0272, October 1981, AD-A109 145.

⁴Michael A. Crombie, *Semiautomatic Pass Point Determination Using Digital Techniques*, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA, ETL-0051, December 1975, AD-A026 082.

⁵Michael A. Crombie and Robert S. Rand, *An Evaluation of the Method of Determining Parallax from Measured Phase Differences*, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA, ETL-0145, December 1977, AD-A056 006.

⁶Michael A. Crombie, *An Evaluation of Conventional Correlation Methods When Matching Infrared Imagery to Panchromatic Imagery*, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA, ETL-0195, August 1979, AD-A076 111.

⁷Michael A. Crombie, *Errors in Automatic Pass Point Mensuration Using Digital Techniques*, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA, ETL-0232, June 1980, AD-A087 443.

⁸Michael A. Crombie, *An Evaluation of Registering Image Gradients When Matching Infrared Imagery to Panchromatic Imagery*, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA, ETL-0250, January 1981, AD-A100 037.

Relaxation of terrain elevation data is another attempt to rescue an automated compilation process from its own weaknesses. This process is a bootstrap operation wherein poorly determined terrain elevations are modified by comparing them with neighboring elevations while taking into account the nature of the terrain. The nature of the terrain in this experiment is characterized by three predetermined slope categories. This study is an evaluation of a relaxation scheme that was developed at ETL in the Computer Sciences Laboratory (CSL) and refined under contract.⁹

The original compilation process is in practice another bootstrap operation wherein little or no information other than the imagery and orientation data is used to control the image-matching process. Local warp information, determined empirically, is used to control the process, and the entire operation depends on the basic notion of correlation. Essentially, correlation describes the linear relation between two sets of data, ignoring all higher order data relationships such as image structure. In fact, if there is a lot of image structure and if the base-height ratio is in the accepted mapping range ($B/H \sim 0.6$), then the correlation process will deteriorate and generally it will break down.

Several lessons have been learned from the work to date. The mapping, charting, and geodesy (MC&G) feature extraction functions cannot be satisfactorily performed in isolation from one another. The necessity for the concurrent extraction of MC&G data has been presented to the mapping community by CSL.^{10,11} This necessity is even more apparent now. Although inclusion of slope data did not remove the larger (in areal extent) errors in this experiment, it is asserted that had the same information been available and used in the original compilation process, the output DTM would have been more acceptable. Such an input to the process can be achieved if the entire MC&G feature extraction is performed in a concurrent and cooperative manner. An operation of this type will require that rules for the interrelationships among different data sets (including known data base information) be developed and imposed upon the process. Such a procedure reflects the notions associated with knowledge bases currently being expressed by experts on artificial intelligence.

⁹Marsha Jo Hannah, *Topographic Relaxation Study*, Final Technical Report, ETL-0209, NASA/Ames Research Center, Institute for Advanced Computation, Moffett Field, CA, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA, September 1979, AD-A095 156.

¹⁰Michael A. Crombie and Lawrence A. Gambino, "Digital Stereo Photogrammetry," Prepared for Congress of the International Federation of Surveyors (FIG), Commission V, Stockholm, Sweden, June 1977.

¹¹Lawrence A. Gambino and Michael A. Crombie, "Manipulation and Display of Digital Topographic Data," Prepared for the Second Symposium on Automation Technology In Engineering Drawing, Monterey, California, November 1979.

NUMERICAL EXPERIMENT

Input Data • Terrain matrices ZZ and ZZN were derived from a stereo pair of digitized scenes, one member of which is shown in figure 1.

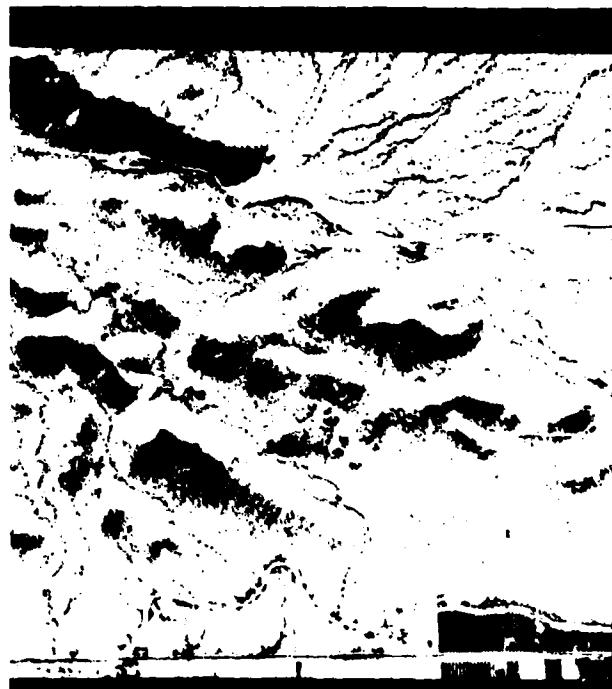


FIGURE 1. Exposure #98.

Each of the DTM's is approximately 400 x 400 pixels wherein the horizontal spacing is about 5.8 meters. The derivations of ZZ and ZZN are described by Crombie and Norvelle.^{12,13} Terrain matrix ZZ was developed in a batch mode before the Digital Image Analysis Laboratory (DIAL) system was in place, whereas ZZN was developed in an interactive mode using DIAL.¹⁴

The superior quality of ZZN over ZZ can be seen by reviewing the gray shade relief images of the two matrices shown in figure 2 and 3.

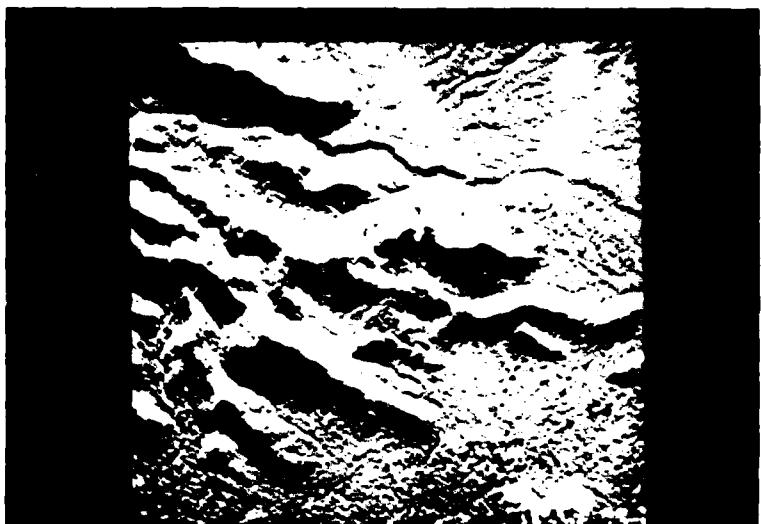


FIGURE 2. Gray Shade Relief of ZZ.

¹²Michael A. Crombie, *Stereo Analysis of a Specific Digital Model Sampled From Aerial Imagery*, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Va., FTL-0072, September 1976, AD-A033 567.

¹³L. Raye Norvelle, *Interactive Digital Correlation Techniques for Automatic Compilation of Elevation Data*, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Va., FTL-0272, October 1981, AD-A109 145.

¹⁴Lawrence A. Gambino and Bryce L. Schrock, "An Experimental Digital Interactive Facility," *Computer*, Vol. 10, No. 8, August 1977, pp. 22-28.



FIGURE 3. Gray Shade Relief of ZZN.

A gray shade relief presentation is derived from a mapping that converts regularly spaced terrain elevations into a low-level terrain surface that is recognizable as such. Essentially, a reflectance model is postulated and reflectance values ($0 \leq R \leq 1$) are computed as functions of local slope, sun angle, and observer. Directions to the fictitious sun and to the observer are chosen for convenience. In both examples the observer was at the zenith and the sun elevation was 20 degrees above the horizontal plane. Local slope is estimated for each position from the DTM. Reflectance values (R) are quantized into 256 gray shades for viewing.

The superior quality of ZZN over ZZ is also demonstrated by the contour maps shown in figures 4 and 5.

REC01 = REC02 ELEVATION DATA

391 X 400

550 = 550 m at 10m interval

ZZ

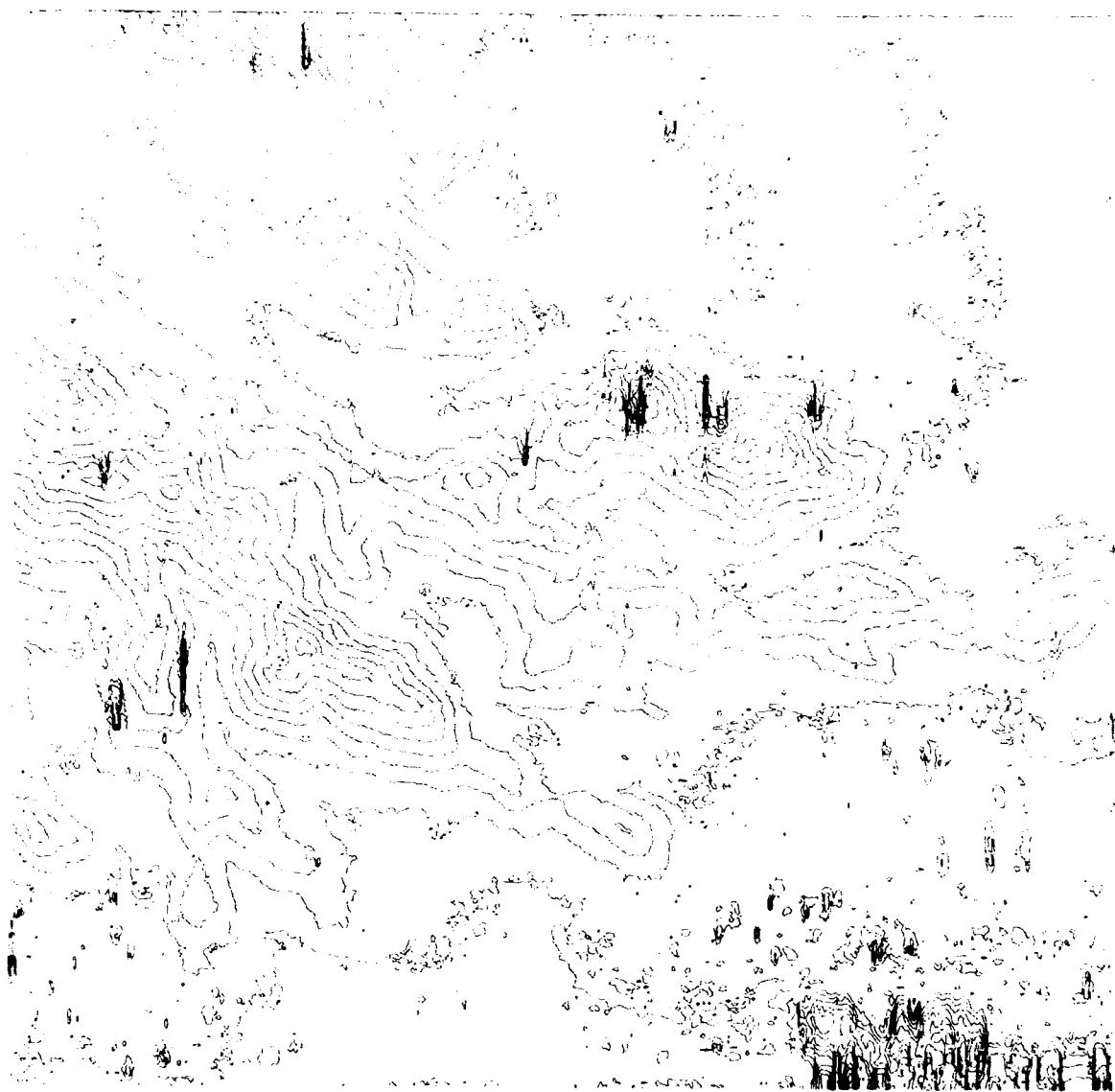


FIGURE 4. Contour Map of ZZ.

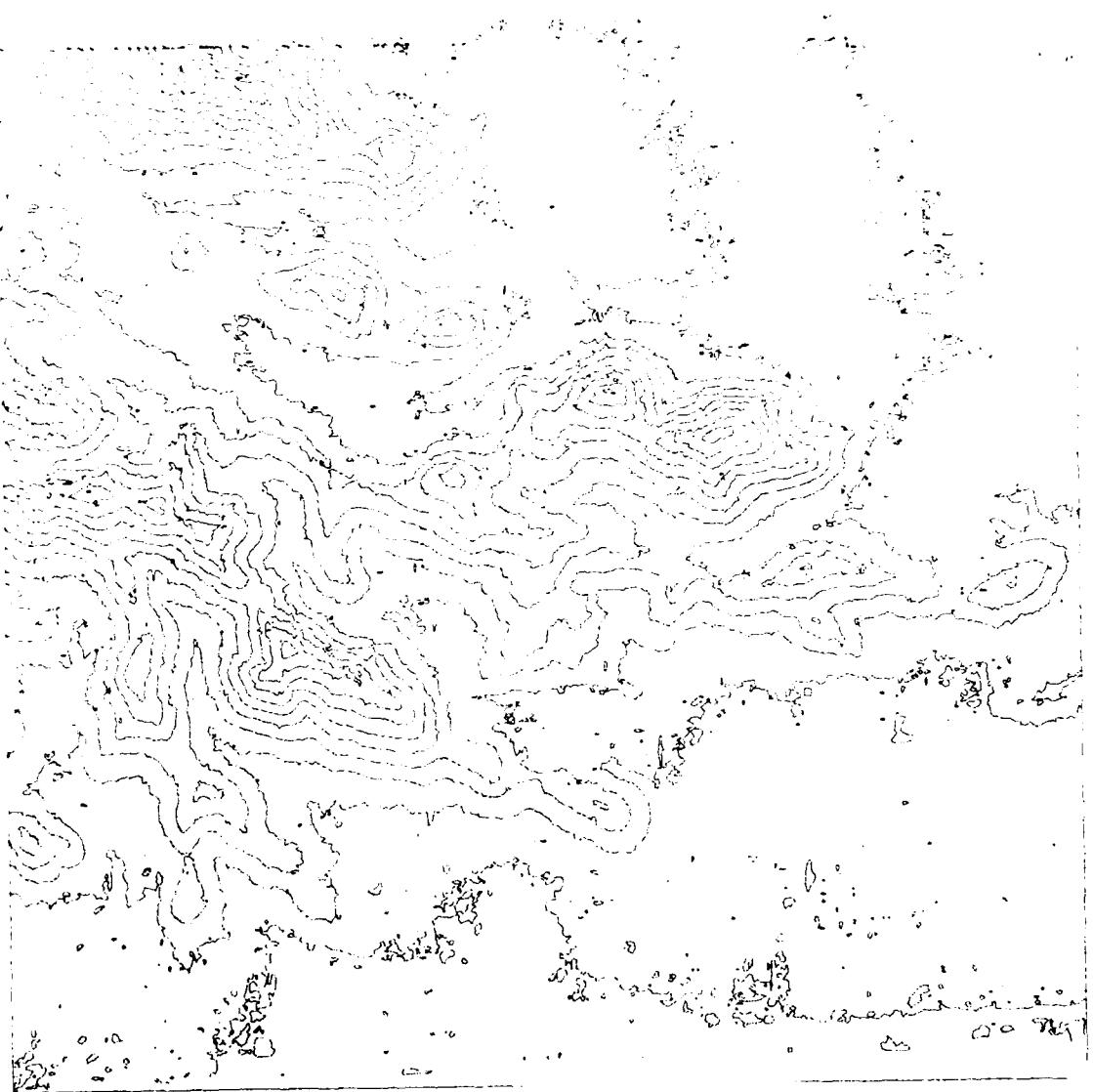


FIGURE 5. Contour Map of ZZN.

The contours are presented at 10-meter intervals. Note how on the ZZ contour map the tracing pen appears to go out of control in portions of the hilly regions and especially in the orchard area in the lower right corner of the picture.

Figure 6 is a slope map derived from ZZN and is described in appendix A. Red pixels refer to flat areas, blue pixels refer to moderately steep areas, and yellow pixels refer to steep areas. The manner in which these data are used as an aid in the relaxation process is described next.

Relaxation Process • Relaxation is an iterative and parallel process wherein local elevation values in the neighborhood of a point are used as evidence for assigning reliabilities to the point and, when needed, to correct the elevation at the point. Evidence is presented in the form of slopes and slope changes at the point in question. These data are calculated using the point in question and its neighbors and assigning a reliability factor to each point before the evidence is reviewed. See appendix A for a discussion on the calculation of elevation reliabilities. The reliability factors are used as weights when elevation corrections are calculated. See appendix B for a discussion on the determination of elevation corrections. A correction at a point is attempted only if the reliability of the point is less than a user-determined value. Generally a computed correction is applied only if the estimated ΔH exceeds a user-determined multiple of the local weighted standard deviation of elevation values. The fundamental quantities that trigger the reliability calculations are user-determined thresholds for the slopes and slope changes.

Input Constraints • Slope and slope change thresholds for this experiment were estimated from ZZ and ZZN. The thresholds were partitioned according to three slope classes, namely, flat (F), moderate (M), and steep (S). The numerical bounds on the three classes are

$$\begin{aligned} F: \quad 0^\circ &\leq \text{Slope} \leq 10^\circ \\ M: \quad 10^\circ &\leq \text{Slope} \leq 25^\circ \\ S: \quad 25^\circ &\leq \text{Slope} \end{aligned}$$

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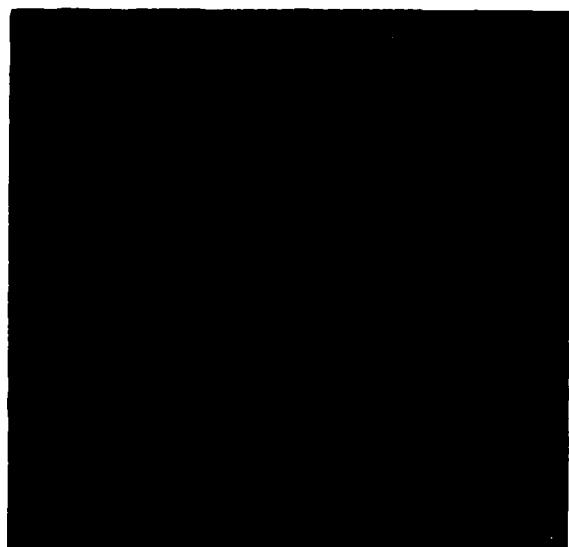


FIGURE 6. SLOPE MAP FROM ZZN

The slope data (SL) and slope change data (DLS and DDS) were calculated from ZZN and organized into cumulative probability estimates from which the 90, 95, and 98 percent thresholds given in table 1 were computed. Similar results derived from ZZ are given in table 2.

TABLE 1. Threshold Values From ZZN

	90%			95%			98%		
	<u>F</u>	<u>M</u>	<u>S</u>	<u>F</u>	<u>M</u>	<u>S</u>	<u>F</u>	<u>M</u>	<u>S</u>
SL	.30	.29	.30	.39	.36	.40	.52	.47	.51
DLS	.25	.25	.25	.35	.34	.35	.57	.48	.51
DDS	.25	.25	.25	.35	.34	.35	.57	.48	.50

TABLE 2. Threshold Values From ZZ

	90%			95%			98%		
	<u>F</u>	<u>M</u>	<u>S</u>	<u>F</u>	<u>M</u>	<u>S</u>	<u>F</u>	<u>M</u>	<u>S</u>
SL	.34	.30	.25	.45	.40	.32	.69	.57	.42
DLS	.31	.28	.24	.47	.40	.32	.90	.69	.47
DDS	.31	.28	.26	.47	.40	.32	.91	.69	.47

The slope data given in table 1 were used to calculate the reliability data given in tables 3, 4, and 5. The tabular entries pertain to the percentage of points that exceed the reliability factor.

TABLE 3. 90 Percent Reliability Thresholds

Reliability Factor	ZZN			ZZ		
	<u>F</u>	<u>M</u>	<u>S</u>	<u>F</u>	<u>M</u>	<u>S</u>
0.95	75.8	49.3	27.6	69.6	60.0	42.8
0.90	90.5	72.7	47.8	85.2	80.1	63.9
0.85	93.7	79.6	57.2	88.8	84.3	70.2
0.80	96.3	86.8	67.5	92.3	90.2	79.3
0.75	97.8	91.2	75.1	94.2	93.1	84.3

TABLE 4. 95 Percent Reliability Thresholds

Reliability Factor	ZZN			ZZ		
	<u>F</u>	<u>M</u>	<u>S</u>	<u>F</u>	<u>M</u>	<u>S</u>
0.95	89.2	69.7	42.7	85.8	80.9	63.1
0.90	96.7	87.3	64.8	92.9	91.9	80.8
0.85	98.0	90.8	72.5	94.6	93.7	84.2
0.80	98.9	94.8	81.3	95.9	96.2	89.9
0.75	99.4	96.7	86.6	96.7	97.2	92.5

TABLE 5. 98 Percent Reliability Thresholds

Reliability Factor	ZZN			ZZ		
	<u>F</u>	<u>M</u>	<u>S</u>	<u>F</u>	<u>M</u>	<u>S</u>
0.95	97.1	88.6	67.3	93.9	94.7	86.5
0.90	99.3	96.5	85.5	96.4	97.4	92.6
0.85	99.6	97.5	89.1	97.2	98.1	94.3
0.80	99.8	98.8	93.7	97.8	98.6	95.7
0.75	99.9	99.3	95.9	98.3	98.9	96.6

A variety of input constraints were imposed upon the relaxation process, and the results were evaluated by reviewing gray shade relief images on DIAL. Results from one of the more successful tests are shown in figures 7 and 8.

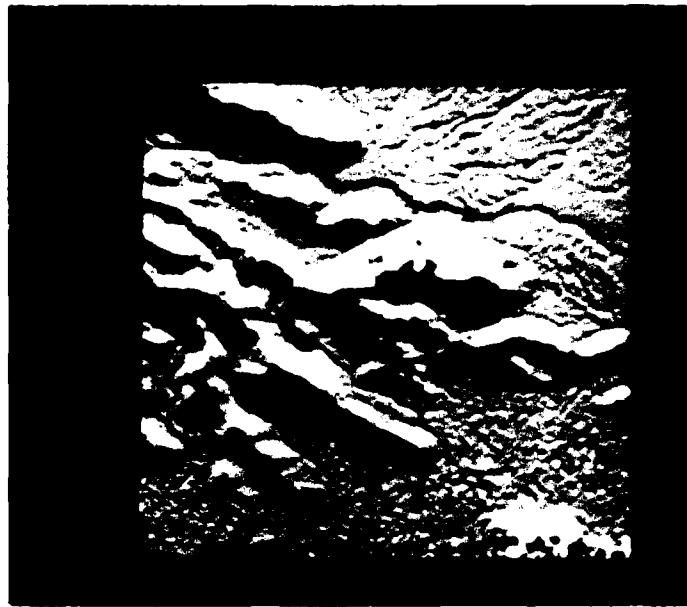


FIGURE 7. Gray Shade Relief of Test Result.

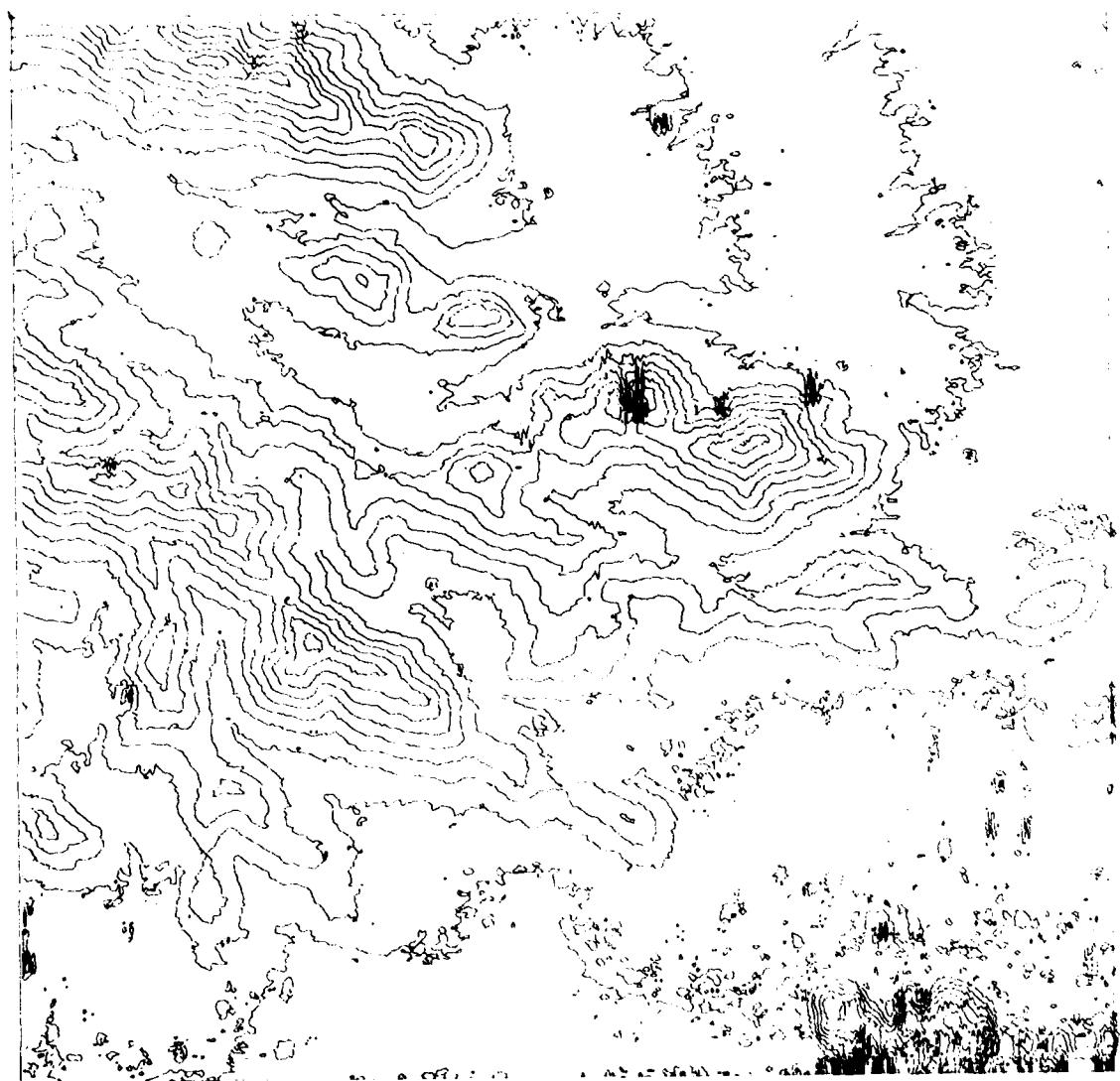


FIGURE 8. Contour Map of Test Result.

DISCUSSION

There seems to be no way of getting around the "garbage in - garbage out" law of data processing. As long as the "garbage" data are few and far between, relaxation methods have an excellent chance of detecting and correcting them. This was shown in a scene classification study wherein probabilistic relaxation was applied to maximum likelihood classification output.¹⁵ There the relaxation process was fairly accurate whenever incorrect classifications were speckled over large regions of correct classifications. Relaxation was also shown to be accurate in a preliminary evaluation of the process described by Hannah.¹⁶ In Hannah's evaluation every eighth line and every eighth value per line of ZZ were processed. The sampling was such that most of the errors were missed. Even in the badly botched areas, such as the orchard, there were sufficient good points so that when slope data were imposed, the process produced reasonable results.

Polynomial approaches either for data compaction or for representing terrain, or both, only compound the problem in that errors are spread over large areas owing to the mathematical process used in generating the polynomial coefficients. Polynomial coefficients derived from another project were used to reconstruct the terrain. The results were displayed on a DIAL work station, and it was readily apparent that much of the terrain had been over smoothed. This may have esthetic value but is unrealistic. At least the relaxation process tends to localize contamination from bad elevation data while the polynomial process tends to spread it over larger areas. In any event, either technique requires preliminary testing to determine how many terms are required and to determine the spacing of points in the polynomial and relaxation approaches. Both parameters are variable; based on the condition of the elevation data.

¹⁵ Michael A. Crombie, Robert S. Rand, and Nancy J. Friend, *Scene Classification Results Using the Max-Min Texture Measure*, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Va., Report in Progress.

¹⁶ Marsha Jo Hannah, *Topographic Relaxation Study*, Final Technical Report, ETL-0209, NASA/Ames Research Center, Institute for Advanced Computation, Moffett Field, Calif., U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Va., September 1979, AD-A095 156.

There are several places on ZZ where the relaxation process seemed to help. Those places where the areal extent of poor elevation was no more than one or two pixels were improved; however, if the incorrect region was on the order of four or five pixels or larger in one direction, then the process tended to produce smooth mounds or smooth depressions. Note in figure 7 the elongated mesas in the orchard area and the large artifact at the mountain peak at the right center portion of the scene. A close look at figure 7 will disclose a large number of smooth terrain dents or mounds that are not readily discerned on the contour map (figure 8).

A close look at the x-parallax of the orchard results on DIAL showed that the batch correlation process (ZZ) went amiss at a building and never recovered. Had the process "known" it was traversing a flat, orchard region, the large set of erroneous output could have been avoided. The interactive process (ZZN), when in the automated mode, encountered the same problems; however, the operator was alerted whenever the correlation was low or whenever an estimated x-parallax seemed excessive. The difficult matches were performed interactively on ZZN, utilizing the floating dot principle in a stereo anaglyph mode.

Basic research is needed to develop procedures to register stereo imagery over regions of little detail, such as bare mountain tops, and at the other extreme, over regions of too much detail, such as a complex of buildings. It appears at this point that the best approach for developing a more reliable, automated match process is one in which more of the MC&G feature extraction is performed in a concurrent manner and rules that express the interrelationships of the features are imposed on the process.

CONCLUSIONS

1. Relaxation is neither an efficient nor a reliable technique for "cleaning up" errors in an elevation matrix.
2. Better match procedures must be developed over image regions of sparse detail and over regions of great detail.
3. The concurrent extraction of all MC&G features in a rule-based process is a promising approach for the automated extraction of terrain elevations, but further refinement is necessary.

APPENDIX A. Elevation Reliabilities

In the relaxation process each point in an elevation matrix was first given a reliability, and then corrections were made on the basis of that reliability. The determination of the reliabilities is described in this appendix, and the actual correction process is described in appendix B.

The measure used to determine reliability was slope. Given any two points, A and B with elevations $H(A)$ and $H(B)$ separated by a distance D , the slope between the two points is $(H(B) - H(A))/D$. More specifically, any point in the matrix has eight immediate neighbors, and eight slope values can be computed (figure A1).

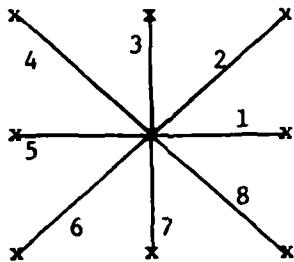


FIGURE A1. Slopes.

In addition to slopes, slope differences or changes may also be computed. From the eight slopes, four local slope changes can be determined (figure A2).

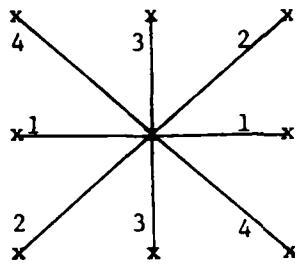
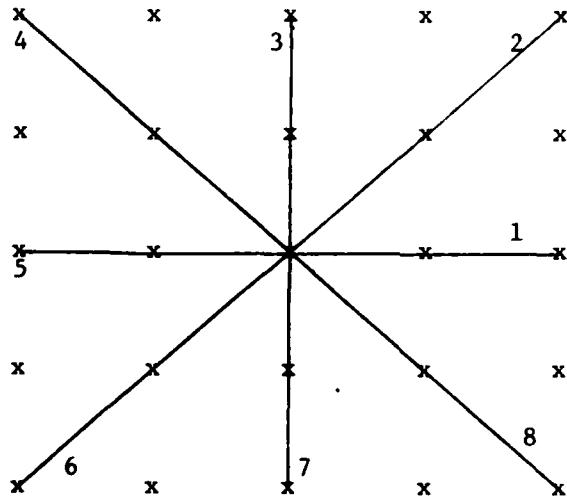


FIGURE A2. Local Slope Changes.

If points two pixels away from the central point are considered, then eight additional distant slope changes can also be computed (figure A3).



Thus for each pixel considered, 8 slope values and 12 slope change values can be obtained.

Two reliability measures were derived for the tests, one for the slope values (RS) and one for the slope difference values (RD). The basic premise for the derivation was to lower the reliabilities if the slope values and slope change values exceeded a certain threshold. The thresholds were determined from a histogram of the ZZ or ZZN values. A variety of thresholds were tested and those associated with the 98th percentile were determined to work best. Separate thresholds were used for slope (DS), local change (DLS), and distant change (DDS). Initial values of RS and RD were determined from the following formulas:

$$RS = 1 - \frac{\sum(\text{slopes over threshold})}{8} \quad (A1)$$

$$RD = 1 - \frac{\sum(\text{local and distant slope changes over threshold})}{12} \quad (A2)$$

Thus a value of 1 would indicate excellent reliability; a value of 0, very poor reliability; and in-between values could be interpreted accordingly.

The thresholds were refined further on the basis of the overall appearance of the land in the part of the matrix being considered. A plane-fitting algorithm was run over the ZZN matrix and this was used to determine the gradient at each pixel. Each pixel was then classified as flat (gradient < 10 degrees), moderate (gradient 10-25 degrees), and steep (gradient > 25 degrees). This classification information was then used in the threshold value selection, resulting in three values each for DS, DLS, and DDS. When the reliabilities were computed, the threshold value for each point depended on its classification. This helped reduce the number of times a steep but valid point was rated low in reliability because it exceeded the overall threshold; conversely, the number of times a flat invalid point was rated high because its thresholds were not high enough was also reduced.

The spacing between the points used to compute the slopes and slope changes was varied. In figures A1 through A3 the immediately adjacent pixels are used (i.e., a spacing of $\Delta = 1$). A spacing of $\Delta = 2$ was also tested. It was thought that the wider spacing would help reduce problems due to small packets of errors, and indeed $\Delta = 2$ did work better than $\Delta = 1$ and it was used in most of the test runs.

After initial reliabilities as described above were determined, these initial values were used to iteratively compute a final reliability. In equation (A1) each point made an equal contribution to modifying the reliability. Now that initial values existed, it made sense to weigh the contribution from each of the surrounding points, based on its reliability. Equation (A1) then becomes

$$RS_{(N E W)} = 1 - \frac{\sum(\text{slopes over threshold}) \times (RS_{(O L D)} \text{ for those points})}{\sum(RS_{(O L D)} \text{ for all eight points})} \quad (A3)$$

Similarly, the contributions for the slope changes were also weighed in computing a new RD. Since two reliabilities were available to choose from (two adjacent pixels for each slope change), the minimum RD for each of these was used.

This process was iterated until the reliability changes became insignificant; three iterations were found to be sufficient. The final reliabilities were then used in the correction process.

APPENDIX B. Elevation Corrections

In appendix A the method of obtaining reliabilities for each elevation point was described. In this appendix the correction process, using those reliabilities, will be discussed.

The reliability process generates two sets of reliabilities, RS and RD; only one set of reliabilities can be used in the correction process. Three options were tried: (1) RS only, (2) RD only, and (3) $\sqrt{RS \cdot RD}$. Options 2 and 3 produced comparable results, both results being better than those from option 1. Since option 2 required less computer time than option 3, it was used in the error correction.

To correct each point, the elevations and reliabilities for the nearest and next nearest pixels in all directions were used, just as in computing the reliabilities (see appendix A). First, the minimum and maximum elevation values in the local region were computed. Then the value between these two extremes was found which minimized S.

$$S = \frac{P + Q}{R} \quad (B1)$$

where

$$P = \sum (\text{local slope changes}) \times (\text{local minimum RS})$$

$$Q = \sum (\text{distant slope changes}) \times (\text{distant minimum RS})$$

$$R = \sum (\text{local minimum RS}) + \sum (\text{distant minimum RS})$$

A quadratic curve-fitting algorithm was used to determine the optimum corrected elevation value.

Not all corrections were implemented; first a correction had to pass a constraint test. The standard deviation σ of the surrounding elevations was computed, and if the absolute value of the correction was less than σ , no correction was made and the old value was retained. Also, no correction was made in the case where the point in question had a reliability greater than .75 (0 = worst, 1 = best).

During experimentation another constraint scenario was tried whereby all points classified as flat with reliabilities less than .75 were corrected, regardless of the value of σ . This was done mainly to help remove the "error mountain" region in the lower right portion of the terrain matrix by ignoring the surrounding values (which were known to be bad); otherwise, these surrounding values would increase the value of σ and possibly prevent a correction when one should have been made.